Teaching Plan: B.Sc. (Hons.) Mathematics, Semester-5

DSC-13: Metric Spaces

Weeks 1 and 2: Definition and examples of metric spaces, Sequences in metric spaces.
[1]: Chapter 1 (Section 1.2 [1.2.1, 1.2.2 ((i), (ii), (iv), (v), (vi), (viii), (ix), (x), (xiv)), 1.2.3, and 1.2.4 (i)], and Section 1.3 [1.3.1, 1.3.2, 1.3.3 ((i), (iii), (iv)), and 1.3.5]).

Week 3: Cauchy sequences, Complete metric space.
[1]: Chapter 1 (Section 1.4 [1.4.1 to 1.4.7, and 1.4.11 to 1.4.14((i), (ii))]).

Weeks 4 and 5: Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem.
[1]: Chapter 2 (Section 2.1[2.1.1 to 2.1.9 (except 2.1.6(ii)), 2.1.12 to 2.1.35, and 2.1.41 to 2.1.44 (except 2.1.42(iv))]).

Week 6: Relativisation and subspaces.
[1]: Chapter 2 (Section 2.2).

Weeks 7 and 8: Continuous mappings, Sequential criterion, and other characterizations of continuity.
[1]: Chapter 3 (Section 3.1 [3.1.1 to 3.1.12, and 3.1.13((i), (ii), (v), (vi))]).

Weeks 9 and 10: Uniform continuity; Homeomorphism, Isometry and equivalent metrics.
[1]: Chapter 3 (Section 3.4 [3.4.1 to 3.4.8], and Section 3.5 [3.5.1 to 3.5.7((i), (ii), (iii))]).

Week 11: Contraction mapping, Banach fixed point theorem.
[1]: Chapter 3 (Section 3.7 [3.7.1 to 3.7.5, except 3.7.2(ii)]).

Weeks 12 and 13: Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings.
[1]: Chapter 4 (Section 4.1 [4.1.1 to 4.1.3, 4.1.4 (statement only), 4.1.5 to 4.1.15]).

Weeks 14 and 15: Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.
[1]: Chapter 5 (Section 5.1 [5.1.1, 5.1.2, 5.1.5, and 5.1.6], Section 5.2 [5.2.1, 5.2.2 (statement only), 5.2.4, 5.2.5, and 5.2.6], and Section 5.3 [5.3.1 to 5.3.8]).

Note: Examples can be discussed in tutorials.

Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

DSC-14: Ring Theory

Weeks 1 and 2: Definition and examples of rings, Properties of rings, Subrings.
[1]: Chapter 12.

Week 3 and 4: Integral domains and fields, Characteristic of a ring.
[1]: Chapter 13.

Weeks 5 and 6: Ideals, operations on ideals, ideal generated by a set and properties, Factor rings, Prime and maximal ideals, Principal ideal domains.
[1]: Chapter 14.

Weeks 7 to 9: Definition, examples and properties of ring homomorphisms; First, second and third isomorphism theorems for rings; The field of quotients.
[1]: Chapter 15.
[2]: Chapter 7 (Section 7.3 [Theorem 7, and Theorem 8((1), and (2))]).

Weeks 10 and 11: Polynomial rings over commutative rings, Division algorithm and consequences.
[1] Chapter 16 (except proof of Theorems 16.2, and 16.3).

Weeks 12 and 13: Factorization of polynomials, Reducibility tests, Mod p Irreducibility test, Eisenstein's criterion, Unique factorization in $\mathbb{Z}[x]$.
[1] Chapter 17 up to Theorem 17.6, page 297 (Theorems 17.3, 17.4, and 17.6 without proof).

Weeks 14 and 15: Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.
[1] Chapter 18 (except proof of Ascending Chain Condition Lemma, and Theorem 18.3).

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
2. Dummit, David S. & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.

DSC-15: Partial Differential Equations

Weeks 1 and 2: Basic concepts, classification, construction, and geometrical interpretation of first-order PDEs.
[1]: Chapter 2 (Sections 2.1 to 2.4).

Weeks 3 and 4: Method of characteristics and general solutions, Cauchy problem for a first-order PDE, Canonical forms of first-order linear equations; Method of separation of variables.
[1]: Chapter 2 (Sections 2.5 to 2.7).

Week 5: Charpit's method for solving non-linear PDEs.
[2]: Chapter 2 (Sections 9 [compatibility condition-based problems only], 10, and 11)

Weeks 6 and 7: Classification (hyperbolic, parabolic, and elliptic), reduction to canonical forms, and general solutions of second-order linear PDEs.
[1]: Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Higher order linear partial differential equations with constant coefficients.
[2]: Chapter 3 (Section 4).

Weeks 10 to 12: Mathematical models: The vibrating string, vibrating membrane, conduction of heat in solids, the gravitational potential, conservation laws and the Burgers equation, Traffic flow.
[1]: Chapter 3 (Sections 3.1, 3.2, 3.3, 3.5, 3.6, and 3.7).
[1]: Chapter 13 (Section 13.6).

Note. For Traffic flow, Chapter 4 (Section 4.8, Problem 4.13.4 and 4.13.5) from the following book may be consulted. Banerjee, Sandip (2022). Mathematical Modeling: Models, Analysis and Applications (2nd ed.). CRC Press.

Weeks 13 to 15: Cauchy problem and wave equations: Solutions of homogeneous wave equations with initial boundary-value problems, and non-homogeneous boundary conditions, Cauchy problem for non-homogeneous wave equations.
[1]: Chapter 5 (Sections 5.1, 5.3, 5.4, 5.5, and 5.7).

Essential Readings

1. Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2. Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

DSE-3(i): Mathematical Data Science

Weeks 1 and 2: Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas.
[2]: Chapter 2, Chapter 3, and relevant material for different presentation styles from Chapter 9.
[1]: Chapter 1 (up to page 28).

Weeks 3 and 4: Anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.
[1]: Relevant material from Chapters 4, 5, and 6.
[1]: Chapter 1 (pages 29- 44, and 58-60).

Week 5: Model driven data in R^n, Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling.
[3]: Chapter 1 (pages 12-13), and Chapter 2 (Section 2.2, 2.3 [2.3.1 to 2.3.3], and 2.4).

Weeks 6 and 7: Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances.
[3]: Chapter 3 (Section 3.3), and Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.
[3]: Chapter 5, and Chapter 6 (Sections 6.1 to 6.3).

Weeks 10 and 11: Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best *k*-rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis.
[3]: Chapter 7 (Sections 7.1 to 7.7).

Weeks 12 and 13: Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for *k*-center clustering, Lloyd's algorithm for *k*-means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering.
[3]: Chapter 8.

Weeks 14 and 15: Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and *k*-nearest neighbors (*k*-NN) classifiers.
[3]: Chapter 9 (Sections 9.1 to 9.5).

Essential Readings

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (https://mathfordata.github.io/).

DSE-3(ii): Linear Programming and Applications

Weeks 1 and 2: Linear programming problem: Standard, Canonical and matrix forms, Geometric solution; Convex and polyhedral sets, Hyperplanes, Extreme points.
[1]: Chapter 1 (Sections 1.1, and 1.3), and Chapter 2 (Sections 2.4, and 2.5).

Weeks 3 and 4: Basic solutions, Basic feasible solutions, Correspondence between basic feasible solutions and extreme points.
[1]: Chapter 2 (Section 2.3), and Chapter 3 (Section 3.2).

Weeks 5 to 7: Simplex Method: Optimal solution, Termination criteria for optimal solution of the linear programming problem, Unique and alternate optimal solutions, Unboundedness; Simplex algorithm and its tableau format.
[1]: Chapter 3 (Sections 3.3, and 3.6 to 3.8).

Week 8: Artificial variables, Two-phase method, Big-M method.
[1]: Chapter 4 (Sections 4.1 to 4.3).

Weeks 9 and 10: Duality Theory: Motivation and formulation of dual problem, Primal-Dual relationships, Statements of the fundamental theorem of duality and complementary slackness theorem with examples.
[1]: Chapter 6 (Section 6.1, and Section 6.2 [up to Example 6.4, and Theorem 6.1 to Example 6.5]).

Weeks 11 and 12: Transportation Problem: Definition and formulation, Northwest-corner, Least-cost, and Vogel's approximation methods of finding initial basic feasible solutions; Algorithm for solving transportation problem.
[3]: Chapter 5 (Sections 5.1, and 5.3).

Week 13: Assignment Problem: Mathematical formulation and Hungarian method of solving.
[3]: Chapter 5 (Section 5.4 [up to 5.4.2 except case study]).

Weeks 14 and 15: Game Theory: Two-person zero sum game, Games with mixed strategies, Formulation of game to primal and dual linear programming problems, Solution of games using duality.
[2]: Chapter 15 (Sections 15.1, 15.2, 15.3, and 15.5).

Essential Readings

1. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). Linear Programming and Network Flows (4th ed.). John Wiley and Sons. Indian Reprint.
2. Hillier, Frederick S. & Lieberman, Gerald J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.
3. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

DSE-3(iii): Mathematical Statistics

Week 1: Joint Distributed Random Variables: Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables.
[1]: Chapter 5 (Section 5.1 up to page 285).

Week 2: Expected values, covariance, and correlation.
[1]: Chapter 5 (Section 5.2).

Week 3: Linear combination of random variables and their moment generating functions.
[1]: Chapter 5 (Section 5.3).

Week 4: Conditional distributions and conditional expectation, Laws of total expectation and variance.
[1]: Chapter 5 (Section 5.4).

Week 5: Bivariate Normal Distribution.
[1] Chapter 5 (Section 5.5 up to page 334 [Regression to the Mean]).

Week 6: Distribution of important statistics such as the sample totals, sample means, and sample proportions, Central limit theorem (statement with examples and applications), Law of large numbers.
[1]: Chapter 6 (Section 6.1 [up to Example 6.3], and Section 6.2 [except Example 6.7]).

Week 7: Chi-squared, *t*, and *F* distributions; Distributions based on normal random samples.
[1]: Chapter 6 (Section 6.3 [Definitions only], and Section 6.4).

Week 8: Concepts and criteria for point estimation, The methods of moments and MLE.
[1]: Chapter 7 (Section 7.1 [up to the Definition, page 408], and Section 7.2 [up to page 423, except Example 7.20]).

Weeks 9 and 10: Assessing estimators: Accuracy and precision, Unbiased estimation, Consistency and sufficiency, The Neyman factorization theorem, Rao-Blackwell theorem, Fisher Information, The Cramér-Rao inequality (statement only), Efficiency.
[1]: Chapter 7 (Sections 7.3, and 7.4).

Weeks 11 and 12: Interval estimation and basic properties of confidence intervals, One-sample *t* confidence interval, Confidence intervals for a population proportion and population variance.
[1]: Chapter 8 (Section 8.1 [up to Example 8.4], Section 8.2 [up to Example 8.9], Section 8.3 [up to Example 8.13], and Section 8.4 [up to Example 8.16]).

Weeks 13 and 14: Statistical hypotheses and test procedures, One-sample tests about a population mean and a population proportion, *P*-values for tests; The simple linear regression model and its estimating parameters.
[1]: Chapter 9 (Sections 9.1, 9.2 [up to page 519], 9.3, and 9.4).
[1]: Chapter 12 (Sections 12.1, and 12.2).

Week 15: Chi-squared goodness-of-fit tests, Two-way contingency tables.
[1]: Chapter 13 (Section 13.1 [up to Example 13.4], and Section 13.2 [up to Example 13.11]).

Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. (3rd ed.). Springer.

Teaching Plan: B.A. (Prog.) with Mathematics as Major, Semester-5

DSC-5: Linear Programming

Weeks 1 and 2: Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity.
[1]: Chapter 2 (Section 2.2).
[1]: Chapter 3 (Sections 3.1, 3.2, and 3.9).

Weeks 3 and 4: Introduction to the simplex method: Optimality criterion and unboundedness, Simplex tableau and examples.
[1]: Chapter 3 (Sections 3.3, 3.4, and 3.5).

Weeks 5 and 6: Artificial variables, Introduction to duality, Formulation of the dual problem with examples.
[1]: Chapter 3 (Section 3.6).
[1]: Chapter 4 (Sections 4.1, 4.2, and 4.3 [Examples 4.3.1, and 4.3.2]).

Note. For weekly plan from weeks 1 to 6, following book may also be followed.
Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.

Weeks 7 to 9: Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problems (Only minimization, balanced and non-degenerate transportation problems to be done).
[2]: Chapter 5 (Sections 5.1, and 5.3).

Weeks 10 and 11: Hungarian method of solving assignment problem.
[2]: Chapter 5 (Section 5.4).

Weeks 12 to 15: Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.
[1]: Chapter 9 (Sections 9.1, 9.3, 9.4, and 9.6).
[2]: Chapter 15 (Section 15.4).

Essential Readings

1. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Teaching Plan: B.A. (Prog.) /B.Sc. (Physical Sc.) with Mathematics, Semester-5

Discipline A-5: Elements of Real Analysis

Weeks 1 and 2: Field and order properties of ℝ, basic properties and inequalities of the absolute value of a real number.
[1]: Chapter 1 (Sections 1.1, and 1.2).

Weeks 3 and 4: Bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of ℝ.
[1]: Chapter 1 (Section 1.6 [1.6.1 to 1.6.14, Theorems 1.6.2 and 1.6.10 without proofs]).
[1]: Chapter 1 (Section 1.5 [1.5.1, 1.5.2, and 1.5.9]).

Weeks 5 and 6: Convergence of a real sequence, Algebra of limits.
[1]: Chapter 2 (Section 2.1).
[1]: Chapter 2 (Section 2.2 [2.2.1 to 2.2.14, Theorems 2.2.8, 2.2.12, and 2.2.13 (d to f) without proofs]).

Week 7: The squeeze principle and applications.
[1]: Chapter 2 (Section 2.3 [2.3.1 to 2.3.14, Theorems 2.3.6, 2.3.10, and 2.3.14 without proofs]).

Weeks 8 and 9: Monotone sequences, Monotone convergence theorem and applications.
[1]: Chapter 2 (Section 2.5 [2.5.1 to 2.5.10, Theorems 2.5.5 and 2.5.7 without proofs).

Week 10: Cauchy sequences, Cauchy criterion for convergence and applications.
[1]: Chapter 2 (Section 2.7 [2.7.1 to 2.7.6, Theorem 2.7.4 without proof]).

Week 11: Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series.
[1]: Chapter 2 (Section 8.1).

Weeks 12 to 14: Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's *n*th root test, Raabe's test.
[1]: Chapter 2 (Section 8.2 [8.2.1 to 8.2.12, 8.2.14, 8.2.15, 8.2.17, 8.2.21, and 8.2.22, with all theorems without proofs]).

Week 15: Alternating series, Leibniz alternating series test, Absolute and conditional convergence.
[1]: Chapter 2 (Section 8.3 [8.3.1 to 8.3.10, Theorems 8.3.2, and 8.3.4 without proofs]).

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Teaching Plan DSE Courses: B.A. (Prog.) with Mathematics, Semester-5

DSE-1(i): Combinatorics

Week 1: Basic counting principles.
[2]: Chapter 5 (Section 5.1).

Weeks 2 and 3: Permutations and Combinations (with and without repetitions), Binomial coefficients, Counting subsets of size *k*.
[2]: Chapter 5 (Sections 5.2 [up to Example 5], 5.3 [up to Theorem 2], 5.4 [Examples 1, and 2], and 5.5 [up to Example 1]).

Weeks 4 and 5: Multinomial coefficients, Set-partitions, The inclusion-exclusion principle and applications.
[1]: Chapter 1 (Section 1.3 [Only Definition], Theorem 1.3.5).
[1]: Chapter 4 (Section 4.1).

Weeks 6 and 7: Generating functions: Generating function models, Calculating coefficients of generating functions, Polynomial expansions, Binomial identity.
[2]: Chapter 6 (Sections 6.1, and 6.2).

Weeks 8 and 9: Exponential generating functions, Recurrence relations: Recurrence relation models.
[2]: Chapter 6 (Section 6.4).
[2]: Chapter 7 (Section 7.1 [up to Example 5]).

Weeks 10 and 11: Divide-and-conquer relations, Solution of linear recurrence relations, Solutions by generating functions.
[2]: [Chapter 7 (Sections 7.2, 7.3, and 7.5).

Weeks 12 to 14: Partition theory of integers: Ordered partition, Unordered partition, Ferrers diagram, Conjugate of partition, Self-conjugate partition.
[1]: Chapter 13 (Sections 13.1.1 to 13.1.9).

Week 15: Durfee square, Euler's pentagonal theorem.
[1]: Chapter 13 (Definition 13.2.1, and Theorem 13.3.1).

Essential Readings

1. Sane, Sharad S. (2013). Combinatorial Techniques. Hindustan Book Agency (India).
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.

DSE-1(ii): Elements of Number Theory

Weeks 1 and 2: Revisiting: The division algorithm, divisibility and the greatest common divisor. Euclid's lemma; The Euclidean algorithm, Linear Diophantine equations.
[1]: Chapter 2 (Sections 2.2, 2.3, 2.4 [up to page 28], and 2.5. All Theorems without proofs).

Weeks 3 and 4: The Fundamental theorem of arithmetic, The sieve of Eratosthenes, Euclid's theorem and the Goldbach conjecture; The Fibonacci sequence and its nature.
[1]: Chapter 3 (Sections 3.1 [Theorem 3.2 without proof], 3.2 [Theorem 3.4], and 3.3 [up to p 53]).
[1]: Chapter 14 (Sections 14.1, and 14.2 [All results without proofs]).

Week 5: Congruence relation and its basic properties.
[1]: Chapter 4 (Section 4.2).

Weeks 6 and 7: Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables.
[1]: Chapter 4 (Section 4.4, Theorems 4.8, and 4.9 without proofs).

Weeks 8 and 9: Fermat's little theorem and its generalization, Wilson's theorem and its converse.
[1]: Chapter 5 (Section 5.2 up to before pseudo-prime at Page 90, Section 5.3 before Theorem 5.5).

Week 10: Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions.
[1]: Chapter 6 (Section 6.1, All Theorems without proofs).

Week 11: The greatest integer function; Euler's phi-function and its properties.
[1]: Chapter 6 (Section 6.3 up to page 118)
[1]: Chapter 7 (Section 7.2, Theorem 7.2 without proof).

Weeks 12 and 13: Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique.
[1]: Chapter 10 (Section 10.1).

Weeks 14 and 15: Introduction to perfect numbers, Mersenne numbers and Fermat numbers.
[1]: Chapter 11 (Sections 11.2 [up to page 223], 11.3 [before Theorem 11.4], and 11.4 [before Theorem 11.10]).

Essential Reading

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

Teaching Plan: DSE courses for B.A. (Prog.) /B.Sc. (Physical Sc.) with Mathematics, and Generic Electives, Semester-5

DSE-1(iii)/DSE-3(ii)/GE-5(ii): Mathematical Python

Weeks 1 and 2: Review of Python fundamentals; Drawing diverse shapes using code and Turtle.
[2]: Chapter 1 (Review: Fundamentals of Python).
[1]: Chapters 1 to 3.

Weeks 3 and 4: Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots.
[3]: Chapter 2 (up to page 45).

- https://patrickwalls.github.io/mathematicalpython/scipy/numpy/
- https://patrickwalls.github.io/mathematicalpython/scipy/matplotlib/
- https://matplotlib.org/stable/gallery/lines_bars_and_markers/index.html
- https://matplotlib.org/stable/gallery/images_contours_and_fields/index.html
- https://matplotlib.org/stable/gallery/subplots_axes_and_figures/index.html
- https://matplotlib.org/stable/tutorials/introductory/pyplot.html

Week 5: Animations of decay, Bayes, Random walk.
[3]: Chapter 5 (Generating random numbers, pages 134 to 139; page 136 is optional).

- https://matplotlib.org/stable/gallery/animation/index.html
- https://matplotlib.org/stable/gallery/animation/animate_decay.html
- https://matplotlib.org/stable/gallery/animation/bayes_update.html
- https://matplotlib.org/stable/gallery/animation/random_walk.html

Week 6: NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace.
[2]: Chapter 4 (pages 226 to 229).

- https://numpy.org/numpy-tutorials/content/tutorial-svd.html
- https://patrickwalls.github.io/mathematicalpython/linear-algebra/eigenvalues-eigenvectors/

Week 7: Solving dynamical systems on coupled ordinary differential equations, Functional programming update fundamentals using NumPy.

- https://patrickwalls.github.io/mathematicalpython/differential-equations/first-order/
- https://patrickwalls.github.io/mathematicalpython/differential-equations/systems/
- https://realpython.com/python-functional-programming/

Weeks 8 and 9: Symbolic computation and SymPy: Differentiation and integration of functions, Limits.
[3]: Chapter 4 (up to page 96), and Chapter 7.

- https://docs.sympy.org/latest/guides/index.html
- https://docs.sympy.org/latest/tutorials/intro-tutorial/calculus.html

Week 10: Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify).

- https://docs.sympy.org/latest/guides/solving/solve-ode.html
- https://docs.sympy.org/latest/tutorials/intro-tutorial/matrices.html
- https://docs.sympy.org/latest/modules/utilities/lambdify.html

Week 11: Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.
[3]: Chapter 4
[2]: Chapter 5.
[1]: Chapter 6, and 10.

- https://docs.sympy.org/latest/modules/solvers/solvers.html

Weeks 12 and 13: Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats.
[3]: Chapter 4 (pages 97-100).

- https://docs.sympy.org/latest/tutorials/intro-tutorial/printing.html

[2]: Chapter 2 (pages 73-83).

- https://pandas.pydata.org/docs/user_guide/io.html

Week 14 and 15: PyLaTeX and writing document files from Python with auto-computed values, Plots and visualizations.

- https://pypi.org/project/PyLaTeX/
- https://matplotlib.org/stable/tutorials/text/usetex.html
- https://pandas.pydata.org/docs/user_guide/visualization.html

Essential Readings

1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978- 1-59327-867-0.
2. Farrell, Peter et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Computer Lab work:

Weeks 1 to 4:

- Spyder Environment preparation with download, installation of required components.
- Implementation of turtle draw for polygonal shapes.
- Using lists and loops for common functions.
- List manipulation.
- Animating objects.
- Interactive grid.
- Drawing complex patterns.

Week 5: Animated plots for solution of problems: decay function w. r. t. time, conditional probability and bayes rule, random walk.
Week 6: Solution of linear algebra problems: Systems of equations, eigenvalues and eigenvectors.
Week 7: Newton's law of cooling, Coupled ODEs with initial conditions.
Weeks 8 and 9: Examples of limits, differentiation and integration of functions.
Weeks 10 to 15: Examples from text references, and similar to those from online sources.

Teaching Plan DSE Courses: B.Sc. (Physical Sc.) with Mathematics, Semester-5

DSE-3(i): Biomathematics

Week 1: Using data to formulate a model, Discrete versus Continuous models, A continuous population growth model.
[1]: Chapter 1 (Sections 1 to 3).
Week 2: Long-term behavior and equilibrium states, Analyzing equilibrium states.
[1]: Chapter 1 (Sections 6, and 7).
Week 3: The Verhulst model for discrete population growth, Administration of drugs.
[1]: Chapter 1 (Section 8).
[2]: Chapter 1 (Section 1.2).
Week 4: Differential equation of Chemical Reactions.
[2]: Chapter 4 (Section 4.4)
Week 5: Predator-prey models (Function response: Types I, II and III).
[2]: Chapter 4 (Section 4.5).
Weeks 6 and 7: Introduction to infectious disease, The spread of an Epidemic: The SIS Model, Interpreting the parameter β, The long-term evolution of the disease, The SIR and SEIR models of an epidemic. [1] Chapter 2 (Sections 1, and 2).
Week 8: Phase plane analysis of epidemic model, Stability of equilibrium points.
[1]: Chapter 2 (Sections 3, and 4).
Week 9: Classifying the equilibrium state; Local stability.
[1]: Chapter 2 (Section 6).
[2]: Chapter 5 (Section 5.4).
Week 10: Limit cycles, Limit cycle criterion and Poincaré-Bendixson theorem (interpretation only with Example 5.6.1). [2]: Chapter 5 (Section 5.6).
Week 11: Bifurcation, Bifurcation of a limit cycle. [2]: Chapter 13 (Sections 13.1, and 13.2).
Week 12: Discrete bifurcation and period-doubling, Chaos.
[2]: Chapter 13 (Sections 13.3, and 13.4).
Week 13: Stability of limit cycles, Introduction of Poincaré plane.
[2]: Chapter 13 (Sections 13.5, and 13.6)
Weeks 14 and 15: Modelling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models; Phylogenetic distances.
[3]: Chapter 4 (Sections 4.4, and 4.5).

Essential Readings

1. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
3. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Note. For an introduction to the SEIR model, refer to Chapter 4 (Section 4.1) of the textbook:
Ellen Kuhl, Computational Epidemiology, 2021, Springer.
(https://link.springer.com/chapter/10.1007/978-3-030-82890-5_4)

DSE-3(iii): Mechanics

Week 1: Fundamental laws of Newtonian mechanics, Law of parallelogram of forces; Equilibrium of a particle, Lamy's theorem.
[1]: Chapter 1 (Section 1.4).
[1]: Chapter 2 (Section 2.2).

Weeks 2 and 3: Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field.
[1]: Chapter 2 (Sections 2.3, and 2.4).

Weeks 4 and 5: Mass centers, Centers of gravity, Friction.
[1]: Chapter 3 (Sections 3.1, and 3.2).

Week 6: Kinematics of a particle, Motion of a particle.
[1]: Chapter 4 (Section 4.1).
[1]: Chapter 5 (Section 5.1).

Weeks 7 and 8: Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity.
[1]: Chapter 5 (Sections 5.2, and 5.3).

Weeks 9 and 10: Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces.
[1]: Chapter 6 (Sections 6.1 to 6.4).

Week 11: Planetary orbits.
[1]: Chapter 6 (Section 6.5).

Weeks 12 and 13: Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity.
[2]: Chapter 1.

Weeks 14 and 15: Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.
[2]: Chapter 2

Essential Readings

1. Synge, J. L., & Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.
2. Ramsey, A. S. (2017). Hydrostatics. Cambridge University Press. Indian Reprint.

Teaching Plan for Generic Electives, Semester-5

GE-5(i): Numerical methods

Weeks 1 to 2: Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.
[2]: Chapter 1 (Sections 1.3.1, and 1.3.2).
[3]: Chapter 1 (Section 1.3).

Week 3 and 4: Bisection method, Secant method, Regula−Falsi method, Newton−Raphson method.
[2]: Chapter 2 (Sections 2.1 to 2.3).
[3]: Chapter 2 (Sections 2.2, and 2.3).

Weeks 5 to 7: Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss−Seidel method.
[2]: Chapter 3 (Section 3.1), Chapter 6 (Sections 6.1, and 6.2), and Chapter 8 (Section 8.1).
[3]: Chapter 3 (Sections 3.2, and 3.4).

Weeks 8 to 10: Interpolation: Lagrange form, and Newton form, Finite difference operators.
[3]: Chapter 4 (Sections 4.2, and 4.3).

Weeks 11 and 12: Numerical differentiation: First and second order derivatives.
[2]: Chapter 11 (Sections 11.1 [11.1.1, and 11.1.2]).

Weeks 13 to 15: Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equations: Euler's method, and Runge-Kutta method.
[2]: Chapter 11 (Section 11.2 [11.2.1, and 11.2.2]).
[1]: Chapter 22 (Sections 22.2, and 22.4).

Essential Readings

1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

GE-5(iii): Elementary Mathematical Analysis

Weeks 1 to 3: Sequential criterion for limits and continuity of functions, Continuity on intervals, Intermediate value theorem and applications.
[1]: Chapter 4 (Section 4.1 [Definition 4.1.1, Theorem 4.1.9, Corollary 4.1.10, 4.1.11, and Example 4.1.12]).
[1]: Chapter 5 (Section 5.1 [Definition 5.1.1, Theorem 5.1.3, Corollary 5.1.4, Example 5.1.5, and 5.1.11]).
[1]: Chapter 5 (Section 5.3 [page 249 to 252, Corollary 5.3.13]).

Week 4: Uniform continuity.
[1]: Chapter 5 (Section 5.4 [up to page 260, first proof that f(x) = 1/x is *not* uniformly continuous on (0,1)]).

Weeks 5 to 7: Riemann integration, criterion for integrability and examples, Integrability of continuous and monotone functions.
[1]: Chapter 7 (Section 7.2).

Weeks 8 and 9: Algebraic properties of the Riemann integral, Fundamental theorem of calculus (first form).
[1]: Chapter 7 (Section 7.5 [Theorem 7.5.1, 7.5.2, and Corollary 7.5.5], alternative independent proofs using Theorem 7.2.14 may be given from Section 33 of K. A. Ross, Elementary Analysis: Theory of Calculus, Springer).
[1]: Chapter 7 (Section 7.6 [Definition 7.6.1, Theorem 7.6.2, and Remark 7.6.3]).

Weeks 10 and 11: Sequences and series of functions: Pointwise and uniform convergence,
Uniform Cauchy criterion.
[1]: Chapter 9 (pages 544 to 551, all theorems without proofs)

Weeks 12 and 13: Weierstrass M-test, Implications of uniform convergence in calculus.
[1]: Chapter 9 (Theorem 9.2.11 to Corollary 9.2.14, page 553]).
[1]: Chapter 9 (Section 9.3 [pages 557 to 562, all theorems without proofs]).

Weeks 14 and 15: Power series, Radius and interval of convergence, Applications of Abel's theorem for power series.
[1]: Chapter 8 (Section 8.6, all theorems without proofs).

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.