SLC (University of Delhi)
Shyam Lal College

**Saraswati IKS Centre**

# Project Title: An Action based Study to Explore & Connect Select IKS in Contemporary Discourses & Practices

# Mathematics in Ancient India and Applications


## Intern Name:

**Dhruv Chokkar,** B.Sc Physical Science with Computer Science, Shyam Lal College

## Mentor Name:

**Dr. Neelam Dabas**, Department of Computer Science, Shyam Lal College

## INTRODUCTION

This research paper is focused on bringing mathematical concepts of Ancient India into light. It is divided into two sections, first one comprising of „Mathematics in Ancient India‟, targeting the mathematical formulae, concepts, values, etc…, and second section comprising „Mathematical Wonders of Ancient India‟, talking about the mathematical concepts being used in construction, rituals, and other great engineering works in Ancient India. We were willing to study manuscripts, such as the Lilavati or Vedic Mathematics, but unfortunately, due to insufficient resources, we had to rely on commentaries available on the internet.

## 1. MATHEMATICS IN ANCIENT INDIA

Mathematics in Ancient India had many concepts which are applicable even in the modern world. Our motive for this section was to study about some important concepts we study today, and how these concepts were studied/taught by our scholars. Arithmetic Progression, Pythagoras theorem, Radius of a Circle, Square root and Cube root, Volume of a sphere are some of the concepts mentioned in this section.

### 1.1 Arithmetic Progression

An Arithmetic Progression (AP) is a series of numbers, with a constant difference in any two consecutive terms. It can be either finite or infinite. In general, we must consider an initial term, which is the first term of an AP; the difference, which is a constant difference between any two consecutive terms and total number of terms, to work with problems related to AP.

Given here is the concept of finding total number of terms in an AP. Given values are the first term of an AP, a, increment/difference, d, sum of all terms, s.

As per modern solution for this question, the formula would go as follows:

$$n = \frac{\sqrt{8sd + (2a - d)^2} - (2a - d)}{2d}$$

However, this method also goes back to time of Brahmagupta, which ranges from c.598 to c.668 CE. He mentioned his method in his work *Brahma-Sphuta-Siddhanta* [1, Sutra 18]. He mentioned the solution in the form of a Sanskrit Shloka, which is:

**उत्तरहीन द्‌धिगुणादी रोषवर्ग धनोतरा ष्टवधे।**

**प्रक्षिप्य पदं शेषोनं द्‌विगुणो तरहतं गच्छ ।।**

To understand the sutra above, we will talk about an example and then translate the sutra, as a part of solution, in the form of steps.

**Example:**

A manufacturing unit produces 20 items on the first day and aims at producing 5 more products than previous day till a total of 150 items are produced.

**Solution:**

**Step 1**: Subtract the increment, d from twice of initial production, a. This would result in remainder, R

**Step 2**: Square the remainder, R and add it to 8 times the product of sum (total production), s and increment, d.

**Step 3**: Take the square root of the result obtained in step 2 and subtract remainder, R from it.

**Step 4**: Divide the result from step 3 by twice of increment, d.

After step 4, we finally get the solution, which is, total number of days in which the target of 150 production will be achieved. On writing steps mathematically, we get:

$$R = 2a - d$$

$$x = R^2 + 8ds$$

$$n = \frac{\sqrt{x} - R}{2d}$$

$$n \; is \; the \; total \; number \; of \; terms \; in \; an \; AP$$

Solving the example with the same method, we get:

$$R = 2(20) - 5 = 35$$

$$x = 35^2 + 8(5)(150)$$

$$x = 1225 + 6000$$

$$x = 7225$$

$$n = \frac{\sqrt{7225} - 35}{2(5)}$$

$$n = \frac{85 - 35}{10}$$

$$n = \frac{50}{10}$$

$$n = 5$$

Hence, the target will be achieved in 5 days, which means, in this AP, the total number of terms is 5. The Arithmetic Progression would be 20,25,30,35,40.

## 1.2 Pythagoras Theorem

Pythagoras theorem states that square on the hypotenuse of a Right-Angled Triangle is equal in area to the sum of squares on other two sides (base and height). We use it many a times to find the length of hypotenuse, which could be a very complex task if we calculate it geometrically. Height and Base of a right-angled triangle can be determined easily, and so, using them, we can find the length of hypotenuse.

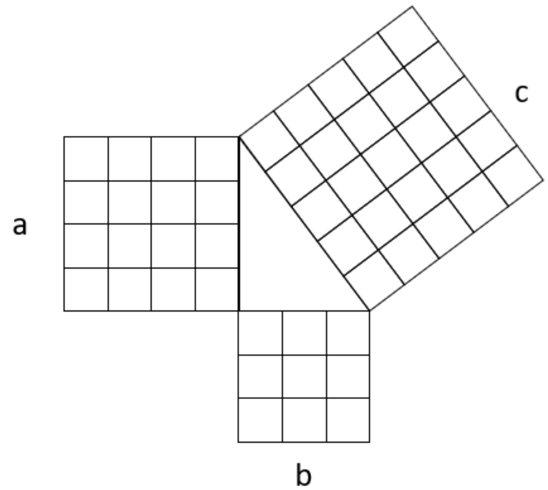



*Figure 1*

According to Pythagoras theorem,

$$a^2 + b^2 = c^2$$

We all are aware of this equation but it is interesting to know that this theorem existed during Ancient Indian times as well. We get to see this theorem in Sulba Sutra: [2]

**दीर्घस्याक्षणया रज्जुः परसवामम , तिर्यडम मम ,**

**च यत्पृथग्भूते कुरूतास्तादुभयं करोति**

*A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together.*

To understand it, we can look at the *figure 1* above and see that total no of squares, with 1 unit sq. area each, at the hypotenuse side, c, is equal to the sum of total no of squares on sides a and b. Hence, a rope stretched along side c with length 5 units will cover an area of 25 unit sq. and will be equal to sum of areas covered by stretching ropes of 4 and 3 units in length, on sides a & b respectively.

That is:

$$5^2 = 4^2 + 3^2$$

$$25 = 16 + 9$$

## 1.3 Radius of a Circle

A radius is the distance between the centre of a circle and any point on its circumference.

A cyclic quadrilateral is a quadrilateral which has all of its four vertices lying on the circumference of the circle. It is also known as inscribed quadrilateral.

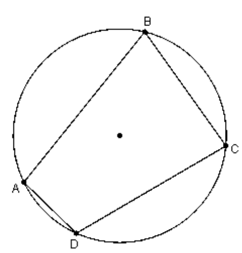
Finding the radius of such a circle (cyclic quadrilateral), when the only known values are the length of all four sides of the quadrilateral, a, b, c & d, can be solved using the formula:

$$Radius, R = \frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

Where,

$$Semiperimeter, s = \frac{a+b+c+d}{2}$$

But in *Yogasraya Commentary*, a commentary written on *Lilavati* [3], two verses gives the solution to this problem. The Sanskrit Shloka (the two verses) is as follows:



**दोष्णां द्वयोर्द्वयोर्घातयुतीनां तिसृणां वधं ।**

**एकैकांनंतरत्रैक्यचतुष्केण विभाजितं ॥**

**लब्धमूलेन यद्वृत्तं विष्कम्भार्धेन निर्मितम् ।**

**सर्वं चतुर्भुजं क्षेत्रं तस्मिन्नेवावतिष्ठते ॥**

On translating, we get:

The three sums of the product of the sides taken two at a time are multiplied together. Divide the equation by the product of four sums of the sides taken three at a time and the fourth one is subtracted. On drawing the circle with radius equal to the square root of the above equation, we get a cyclic quadrilateral.

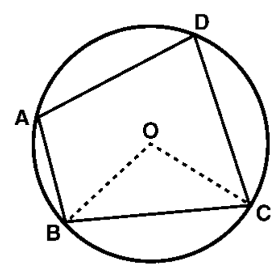
It may be difficult to understand this just by reading, so I tried to formulate this shloka and convert it into a simpler version, written in the form of a mathematical formula.

The formula is as follows:

$$Radius = \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(a+b+c-d)(b+c+d-a)(c+d+a-b)(d+a+b-c)}}$$

Where:

$$AB = a$$
$$BC = b$$
$$CD = c$$
$$DA = a$$




Let us now verify the research using an example.

Let the sides of the cyclic-quadrilateral be 10, 20, 30 & 50 cm respectively.

Using modern method:

$$s = \frac{10 + 20 + 30 + 50}{2} = \frac{110}{2} = 55$$

$$R = \frac{1}{4}\sqrt{\frac{(200 + 1500)(300 + 1000)(600 + 500)}{(55 - 10)(55 - 20)(55 - 30)(55 - 50)}}$$

$$R = \frac{1}{4}\sqrt{\frac{1700 \times 1300 \times 1100}{45 \times 35 \times 25 \times 5}}$$

$$R = \frac{1}{4}\sqrt{\frac{2431000000}{196875}}$$

$$R = \frac{1}{4}\sqrt{12347.94}$$

$$R = \frac{1}{4} \times 111.12$$

$$R = 27.78$$

Now using the method given in *Lilavati* which we have explained earlier in this section,

$$R = \sqrt{\frac{(200 + 1500)(300 + 1000)(600 + 500)}{(10)(90)(70)(50)}}$$

$$R = \sqrt{\frac{1700 \times 1300 \times 1100}{3150000}}$$

$$R = \sqrt{\frac{2431000000}{3150000}}$$

$R = \sqrt{771.74}$

$R = 27.76$

We found that results from both the methods are approximately same. It is also noted that where we found the radius from modern method in 7 steps, we were able to determine the radius in just 5 steps from Ancient Indian method, that too, without requiring to find additional value of Semi perimeter, s.

## 1.4 Square Root and Cube Root

A square root of a number is the value, which on multiplying by itself, gives the original value. And a cube root of a number is the value, which on multiplying by itself taken 3 at a time, gives the original value.

To get the square root or cube root is one of the most hectic tasks a student finds if he has to calculate it manually.

Thanks to the *Yogasraya Commentary,* written on the *Lilavati* [3], for discovering an easy way to calculate square root and cube root of any number.

The method is written in Malayalam, but rendered in English for easy understanding. It is:

*oru vargasamkhye milikkenam ennu varikil á vargasamkhye veccu istamayiṭṭu oru sankhye harakamayi kalpiccu mite veccu harippi | haricca phalatte harukat- til kütti arddhippu atine kontu pinneyum natette vargasamkhye tanne harippu pinneyum atukontu vargasamkhye harippi | i vannam hárakattinnu avišesam naru- volam ceyyu ennal a harakam tanne à vargasankhyete milamennrika | pinne ghanatte mülippanum ghanasamkhye veccu i vannam vallatum oru harakatte kalpiceu harippu hariccuptaya phalatte pinneyum á hárakam kontu tanne harippu! avile untaya phalatte härakattil kütti ardhippu pinne atine harakamayi kalpiccu harippi avisesam varuvolam ennal ghanamulavum varum* | [3]

~Malayalam Language written using English alphabets

Translation:

Divide the number we want to find the square root of by any number. Then find the average of the divisor and quotient. Ignore the remainder in case of average not a whole number. Repeat the process, treating this average as the new divisor till you do not get a remainder. At this stage the divisor and quotient will be the same, which is the square root.

For the cube root, divide the given number by any chosen number. Then divide the quotient obtained again by the same chosen number, neglecting the remainder.

Then find the average of the divisor and quotient. Repeat the process, treating this average as the new divisor till the remainder is not obtained. The quotient will be the cube root.

Let us now find out the square root of 81 using this method.

Divide 81 by any number, say 3. We get 27 as the quotient. Now find average of quotient (27) and divisor (3). We get 15 as the average. Not dividing 81 by the new divisor, 15, we get quotient 5 and a remainder 6. Ignoring the remainder, we will again find the average of quotient and divisor, i.e., 15 & 5. It would result in 10. Now dividing 81 by 10, we get 8 as the quotient and 1 as the remainder. Ignoring remainder again, we will find average of 8 and 10, we get 9. Divide 81 by 9 and we get quotient 9 and remainder 0. Hence, 9 is the square root of 81.

Similarly, we can find cube roots of the number with one additional step of dividing new quotient as well with the divisor.

### 1.5 Area of a Circle and Volume of a sphere

Area of a circle equals to pi times the square of its radius. Whereas, volume of a sphere is 4/3rd of pi times the cube of its radius. That is:

$$Area\ of\ a\ circle = \pi r^2$$

$$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

Though these methods are mathematically correct, we also find another method of finding the area of a circle and volume of a sphere in *The Abhipreta Commentary* [3]. A verse is mentioned below, which formulates the solution to find the area and the volume.

**व्यासस्य वर्गाच्छरसायकाग्निक्षुण्णाद् द्विपञ्चाब्धिहृतं फलं स्यात् ।**

**घनीकृतव्यासक एवं नागाद्रयाहतं गोळफलं घनाख्यम् ॥**

It is translated as follows:

The area of a circle equals to product of the squares of its diameter and the number 355, whole divided by 452. Also, the volume of a sphere is obtained by multiplying the cube of diameters with 355 and dividing the result with 678.

Let us suppose that „D" is the diameter in both the cases, i.e., in case of a circle and a sphere. According to the verse, we get to know that:

$$Area\ of\ a\ circle = \frac{355 \times D^2}{452} \qquad Eq. 1.$$

$$Volume\ of\ a\ sphere = \frac{355 \times D^3}{678}$$

It may also be noted that this verse gives a better approximation of the constant value, which in modern world, is known as Pi. To get the value of this constant, or Pi, we can see the following method:

$$Diameter, D = 2 \times Radius, R$$

Now putting R instead of D in equation 1, we get:

$$Area\ of\ circle = \frac{355 \times (2R)^2}{452}$$

$$Area\ of\ circle = \frac{355 \times 4R^2}{452}$$

On solving for $\frac{355\times4}{452}$ as a constant value, we get 3.1415929203…. which is almost like the modern calculated value of Pi.

Hence, we observed that how beautifully one concept is linked with other in Ancient Indian Mathematics.

## Applications of Mathematics in Architecture & Dance

### 1. Konark Sun Temple [4]

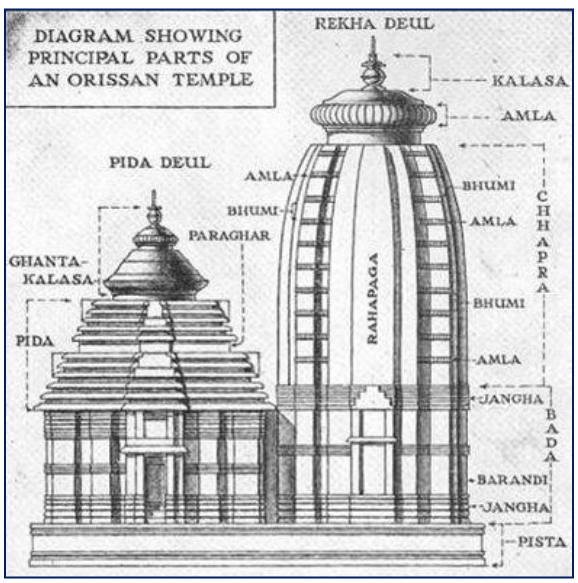
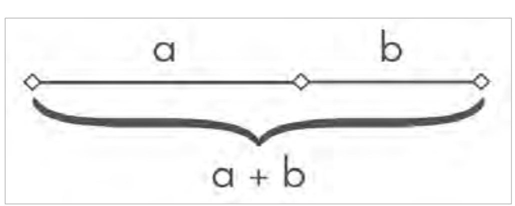



*Figure 2 [5]*

A Mathematical Wonder – Konark Sun Temple is situated in Odisha, India. With measurements as precise as generated by computers, and genius minds behind its architecture and engineering, the Konark Sun Temple still stands as the icon of Bharatiya Knowledge System. Let us look more into the temple in mathematical terms.

### <u>Golden Ratio</u> [6]



“The golden ratio, also known as the golden number, golden proportion, or the divine proportion, is a ratio between two numbers that equals approximately 1.618. It is strongly associated with the Fibonacci sequence, a series of numbers wherein each number is added to the last.”

Architects believe that a rectangle with sides in Golden Ratio looks aesthetically most pleasing to the eye, this ratio, in context to the image above, is as follows:

$$\frac{a+b}{a} = \frac{a}{b} = Golden\ Ratio$$

Konark Sun Temple, a mathematical wonder in Konark Town of Odisha, which is about 35 KM northeast from Puri City. Its main complex is comprised of two parts, a main temple and an assembly hall. Unfortunately, the 229 ft tall Shikhara, in the temple premises is not present today because of its destruction in 1837. But the assembly hall is remains standing today as well. Another surviving structure of the temple is 128 ft tall Jaganmohan, which is also one of the principal structures. Dimensions of the Jaganmohan are 857 ft by 540 ft, which is also the mathematical wonder we are talking about. These dimensions result in the Golden Ratio of the Konark Sun Temple to be 1.58, which is very near to the ideal value, which is 1.618.... Hence, it is an excellent example of usage of Golden Ratio in Ancient Indian Architecture making it pleasing to eyes.

**Directions**

The sun temple is built in such a way that first rays of the sun directly fall in the Garbha-Grah, which signifies the usage of trigonometry in ancient India. It also has a set of 24 wheels carved in the temple complex which indicates us about the 24 hours in a day. The sundial is again a masterpiece in this temple, where it is so precise and accurate in telling the time of a day as close to a minute. It is also amazing to know that sundial at Konark sun temple works anti-clockwise. This is a wonderful piece of work of geometry. The sundial is explained mathematically below:

***8 major spokes:***

*Figure 3 [7]*

These spokes divide a day, or 24 hours, into 8 equal parts. Which means, each part defines 3 hours or 180 minutes. Angle between each major spoke is 45 degrees.

***Minor spokes:*** Minor spokes, which can be seen in the picture, dividing each pair of major spokes into half. This further divides 3 hours into two equal parts, comprising 90 minutes in each part. Angle between each pair of the minor spoke and the major spoke is 22.5 degrees.

***Beads***

On observing carefully, we can see a lot of beads at the edge of the wheel. There are 30 beads in each pair of a major spoke and a minor spoke. Which hence, divides 90 minutes into 30 equal parts, 3 minutes each. The beads are large enough, so that we can see the shadow falling on the dial, between these beads, accurately and tell which part of the area between the beads the shadow is falling. Hence, we can visually divide the beads into 3 parts, one minute each. Angle between each pair of beads is 0.75 degrees. It is amazing that the engineers of ancient India could calculate and constructing something so precisely.

Hence, it is concluded that the sundial at Konark sun temple is so precise and accurate that a person can tell the time of a day as close to a minute.

## 2. Yagya Kunds in Ancient India [8]

In ancient times in India, while performing Yagya, people had to be so precise and accurate in its construction so that the yagya is successful. Be it for religious approach or mathematical approach, they proved that precise shapes can be formed without using smart tools. It is a fact that in many writings, it is mentioned that they used just Bolts and a Rope to construct Yagya Kunds.

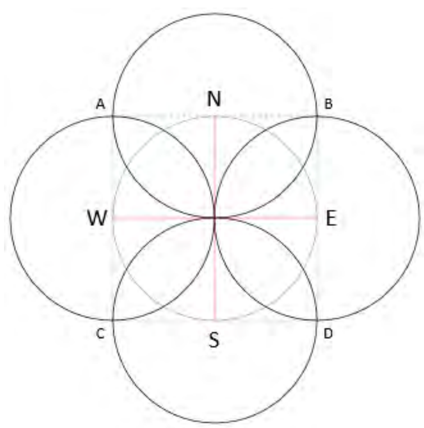
Below is the explanation of few concepts which were used in Yagya Kunds back at that time.

**Volume**

Yagyas were performed in such a way that the Kund in which offerings were presented, should neither overflow with the offerings neither get filled to the top. It was to be kept in mind that the Kund only fills up 2/3$^{rd}$ of its actual volume. And to achieve this, it was necessary to know the concepts of calculating volume and even surface areas.

Also, two kunds of different shapes had to be of same volume. Here comes the concept of converting the square to a circle of same area as that of square and vice versa.

**<u>Perfect Square using Baudhayana Sulba Sutra</u>**




It is the concept of constructing a perfect square with use help of rope and bolts.

1. Put two bolts on west and east respectively.

2. Determine the centre of this line by constructing the perpendicular bisector which will eventually be directed towards North and South.

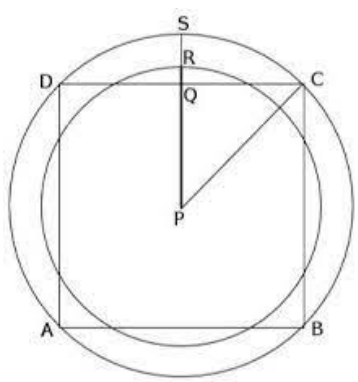
3. With intersection as the centre and Centre to either West or East radius, construct a circle around the centre.

4. Now with the intersection points of lines and the circle, draw 4 circles, each having the radius of one of the intersection points.

5. Now the intersections of these four circles resembles the vertices of a square which is so precise and accurate.

**<u>Circling the square with same area as that of square</u>**

Here is the concept of conversion of shapes with same area, with the use of only bolts and ropes.




1. Determine the centre of square and construct a line from that centre to any one vertex of the square.
2. With this line as the centre, draw a circle around the square.

3. Now determine the centre of any one side of the square and construct a line from centre of a square to the circle edge, passing through centre of the side.
4. Now mark $1/3^{rd}$ of the total length from circle edge to the side of a square.
5. Take radius of centre of square to this point ($1/3^{rd}$) and draw a circle.

Hence, desired circle is obtained, which is having area same as that of a square.

3. **Mathematics in Dance – Odissi Dance** [9]

Odisha, an eastern coastal state of India. Here lies a very rich heritage of Odissi Dance. This dance form not only showcases the dance as many others, but it is a very beautiful blend of mathematics, more precisely, geometry and postures which tells a story, a poem, or even spiritual ideas through dance. We will now be mentioning some of the dance forms and postures which today‟s technology was able to extract data about.

The three primary dance positions in Odissi are:

**Samabhanga**

With straight spine and arms raised up with slight elbow bent, the weight of the body is distributed equally on both the legs. As seen in the figure, we can observe that centre of the square around the body lies at centre of the human body and arms and legs together resembles a square. It is also believed that this pose provides the serenity to the soul as the body stands so symmetric that there is no pressure on any part of the body.

**Abhanga**

With deep leg bends and feet & knees turned outwards, the dancer experiences a weight shift from one leg to another. And during the movements, we can also observe the movement of hips, indicating the change in centre of mass of the body which is the reason behind weight change. It resembles meditation.

**Tribhanga**

Tri-Bhanga, meaning three breaks. Evolved in such a way that one half of the lower body remains static along the central plumb line whereas the other leg crosses the first leg. Also, upper half of the body from the torso is deflected in the opposite direction. Third deflection comes from head or neck. The Tribhanga is designed by deflecting the hip from the horizontal Kati Sutra with head deflecting on the same side as that of the hip. Also, the knees, the torso and the neck are bended in this dance form.

**Conclusion**

I would like to conclude that Mathematics in Ancient India was an extraordinary achievement of that time. With no computers and precise instruments as those of today, our Ancient Mathematics Scholars were very well able to determine several theories, concepts of mathematics and work on them. They also described several formulas, rules in the form of Shlokas, which were quite easy to remember, understand and implement. Also, it was interesting to know that how they used daily use things like ropes in proving mathematical concepts. With this, I conclude my research journey on Ancient Indian Mathematics, aiming to gather more knowledge, with more accuracy in upcoming time. Mathematics is not only a thing of calculation on papers or high-level scientific knowledge. It can, as we saw in this research papers, also be applied in various other fields like architecture, dance, rituals and much more. It"s the best language to define and design any art form. Ancient Indian scholars not only explored the wide range of mathematics applications in real world, but also gave a futuristic vision to all the upcoming scholars, students about maths. They proved that Maths can be a way of living our lives.

**References**

1. Sharma, A. R. (Ed.). (1966). Brahma-Sphuta-Siddhanta with Vasana, Vijnana and Hindi Commentaries (Vol. I and III). Bangalore, Karnataka: Indian Institute of Astronomical Research.
2. https://www.booksfact.com/science/ancient-science/pythagorean-pythagoras-theorem-in-baudhayana-sulba-sutra-200-bce.html
3. https://www.academia.edu/38226643/A_Study_of_Two_Malayalam_commentaries_on_Lilavati

4. Garima Yadav (2021), *The Konark Sun Temple: Mathematics Behind The Architecture of An Astronomical Wonder*, International Journal of Recent Scientific Research Research, Vol. 12, Issue, 01 (D), Pp. 40690-40692.

5. https://www.archinomy.com/wp-content/uploads/case-studies/2011/deul-jagamohan-parts.jpg?ezimgfmt=rs:390x612/rscb2/ngcb2/notWebP

6. https://www.adobe.com/creativecloud/design/discover/golden-ratio.html

7. https://i.pinimg.com/originals/df/20/a2/df20a2c7b5c55f66a139779ffea0eec0.jpg

8. https://www.youtube.com/watch?v=s723-3hkUjA

9. Tarini Anchan (2017). *Studying Traditional Dance Forms -Mathematical Study of Odissi Dance Postures*, International Journal of Advance Research In Science and Engineering, Vol. 6, Issue 07

10. Vatsyayan Kapila -The Square and The Circle of Indian Arts, (105)